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Reliability Assessment of Interdependent Lifeline Systems (RAILS) and Systemic Importance Measures Using a Non-Simulation Method

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Abstract: Lifeline Systems (LSs) are the physical and cybernetic distributed networks that underpin modern societies. They include: power grids, transportation networks, cybersecurity networks, and finance networks, among others. As urban populations continue to develop, LSs need to expand in capacity and coverage to sustain ever increasing loads. Ensuring a reliable and steady supply of commodities and services from LSs is among governments' and all other stakeholders' best interests. However, assessing the reliability of LSs and finding cost-effective strategies to improve their resilience remain standing challenges due to the computational hardness of methods for evaluating interconnected LSs. This paper proposes a new reliability estimation method inspired by previous work on set theoretic methods for estimating network reliability. Moreover, we augment the application of existent importance measures to uncover cascading failure vulnerabilities across and among LSs, which are deemed practical to inform resource allocation and LS asset management.

1 Introduction

The reliability and performance assessment of lifeline systems (LSs) continues to be an active area of research and a crucial aspect to consider in frameworks and methodologies for the study of resilience of LSs [5, 19]. However, whenever complexity arises from the scale of systems or inclusion of more constraints, such as system demands and capacities, the possibilities for practitioners and researchers narrow to simulation methods for performance assessment. One of the main limitations of simulation methods is the difficulty of determining the precision of estimations, or that in order to do so, one requires to conduct large computational experiments [10]. Furthermore, machine learning algorithms remain problem-specific as well as site-specific. To overcome these limitations, alternatives emerge from a combination of analytical and sampling methods that either estimate exactly, within theoretical bounds, or within confidence intervals the reliability of infrastructure systems. An emerging approach for computing reliability of LSs is based on the state-space-partition (SSP) method [15, 3], proven computationally efficient for network reliability problems. Such techniques partition the space of possible states for a given system into subsets of feasible and infeasible states. For example, link-sets (feasible) and cut-sets (infeasible) in connectivity problems. This approach was first developed for multi-state systems by Doulliez and Jamoulle [9] in the context of feasible flow problems. Later on, Alexopoulos found mistakes and corrected them [2], and it was further applied for stochastic spanning trees and multi-terminal flow problems [15, 7]. Furthermore, considering that flow problems can be reduced to path problems, the decomposition principles in the work by Dotson [8] is a special case of multi-states systems when they are reduced to the binary case. We make this observation given that the work by Dotson is the basis for efficient methods that were more recently developed in the reliability and earthquake engineering community (e.g. [17]). In addition to this, the fist attempt to study interdependent LSs using non-simulation methods was conducted by Kim, Kang and Song [16] in which they studied dependencies of water nodes on the power system using graph transformations such as super nodes and connecting the power and water networks in series. However, the previous work does not consider the bidirectional case of interdependencies among LSs, which is more challenging and is essential to shed light on the vulnerability of interdependent LSs. This paper presents a new method for reliability assessment of interdependent lifelines systems that fully supports interdependencies among systems. The core of our contribution consists of developing a state-space-partition SSP technique for systemic Reliability Assessment of Interdependent Lifeline Systems (RAILS) problems. The outputs of this method not only provide reliability estimates, but also reveal cascading failure vulnerabilities via importance measures for LSs' components. The reminder of this paper is structured as follows. Section 2 describes the new method for the estimation of systemic reliability of interdependent LSs as well as the extension of importance measures to the system-of-systems case. Section 3 shows computational experiments for a set of synthetic networks studied using RAILS. Finally, Section 4 offers conclusions from this study and outlines future research directions.

2 Reliability Assessment of Interdependent Lifeline Systems

This section begins by introducing the network reliability problem and its generalization to the system-of-systems case. Later on, we introduce a new SSP method and it's application to the feasible flow of interconnected systems. At the end of this section we will show extensions of importance measures in the literature to the systemic network reliability case.

2.1 Network Reliability

Assume that an infrastructure network k can be modeled as graph $G_k(V_k, E_k)$, where V_k is the set of nodes and E_k is the set of links such that $|V_k| = n_k$, $|E_k| = m_k$. In practice, links and nodes represent physical assets such as transmission lines and telecommunication towers. Both, nodes and links (network components), can fail or vary their capacity levels. Assume an arbitrary labeling of components $\mathscr{L} = \{1, 2, .., a\}$ such that $a = n_k + m_k$. We can represent mathematically such labeling by using one-to-one mappings $\psi : V_k \mapsto \mathscr{L}$ for nodes and $\phi : E_k \mapsto \mathscr{L}$ for links. Furthermore, we can model the capacity level u_i of a component $i \in \mathscr{L}$ as a discrete random variable taking values from the finite set $\{x_i(1) < ... < x_i(l_i)\}$ with respective probabilities $\{p_i(1), ..., p_i(l_i)\}$, with l_i representing the total number of states considered for component i. The probabilities $p_i(j)$ represent the probability mass function (PMF) of component's i capacity levels. Such PMFs are discretized versions of component fragilities, which are usually obtained from expert opinion, empirical models derived from observations, and computational models. Moreover, we can represent the stochastic state of the system with a vector $X(v) = \{x_1(v_1), ..., x_a(v_a)\}$, where v_i represents the index of component's i capacity level. We can use indifferently $X(v)$ or $v = \{v_i, ..., v_a\}$ to represent the state of the infrastructure system. We can define the state-space of the system Ω as the cross product:

$$\Omega = \prod_{i \in \mathscr{L}} \{1, ..., l_i\} = \{v = (v_1, .., v_a) : 1 \leq v_i \leq l_i, \forall i \in \mathscr{L}\} \tag{1}$$

Typically, $x_i(1)$ and $x_i(l_i)$ represent complete damage and no-damage, respectively. Thus, the system's ideal state is verified when $v = \{l_i, ..., l_a\}$ and it will perform at its worst when $v = \{1, ..., 1\}$. It is important to note the hyper-rectangular structure of Ω. This structure has been exploited by many set-theoretic decomposition algorithms to estimate network reliability of binary systems. Furthermore, the generalization of the binary formalism is the multi-state system case, where set theoretic decomposition methods are often referred as state-space-partition (SSP) methods [2]. Regardless of the infrastructure system under consideration, we can value its performance at any state $v \in \Omega$ using an infrastructure utility function $u_k(v)$ linked to a performance metric [14]. Thus, by setting a desired performance threshold D_k, we can define the system's infeasible domain as $\mathscr{I} = \{v : u_k(v) < D_k\}$ and its feasible domain as $\mathscr{F} = \{v : u_k(v) \geq D_k\}$. In practice, $u_k(v)$ can measure the number of served costumers by a power distribution network, or an inverse relation to customer disconnections and travel times in telecommunication networks and transportation networks, respectively. We can define network reliability $r(G_k)$ of the stochastic system G_k as follows:

$$r(G_k) = \Pr[v \in \mathscr{F}] = 1 - \Pr[v \in \mathscr{I}] \tag{2}$$

Estimating r is an NP-Hard problem [23, 21], which means that an algorithm that estimates r exactly runs in exponential time as a function of the size of the input. Thus, approximation algorithms and sampling schemes have been proposed to obtain estimates of r that are useful in practice [13, 6, 11]. Also, Markov Chain Monte Carlo methods have been used for the network reliability problem, but without guarantees in the quality of approximation [24]. In any case, approximation and sampling algorithms can require a prohibitive number N of $u_k(v)$ evaluations. In general, the problem of evaluating $u_k(v)$ can be casted to a mathematical optimization problem that is often difficult to solve as a becomes large. Thus, methods that keep the number of $u_k(v)$ evaluations to a minimum and offer guarantees of approximation while scaling to real-world applications are in high-demand.

The following subsections generalize the single infrastructure formalisms discussed above to the system-of-systems case and introduce a new method for estimating the reliability of interconnected systems.

2.2 Systemic Reliability Assessment of Interdependent Lifeline Systems (RAILS)

LSs are part of the built environment that is pillar to the well-being of communities. As such, an accurate model needs to consider their combined performance to contingencies in the face of community needs and social expectations [18]. We begin this section by extending the previous notation to the system-of-systems case. Consider a set of infrastructures K, each composed of stochastic systems $G_k(V_k, E_k)$ with set of nodes V_k and set of links E_k for all $k \in K$. The ensemble of stochastic systems is denoted $G(V,E)$, with set of nodes $V = \bigcup_{k \in K} V_k$ and set of links $E = \bigcup_{k \in K} E_k$. As before, let us consider an arbitrary labeling of components $\mathscr{L} = \{1, ..., a\}$ with $a = |V| + |E|$, and one-to-one mappings $\psi : V \mapsto \mathscr{L}$ for nodes and $\phi : E \mapsto \mathscr{L}$ for links. Also, component capacity levels u_i for all labeled components $i \in L$ are modeled as discrete random variables taking values from the finite sets $\{x_i(1) < ... < x_i(l_i)\}$ with respective probabilities $\{p_i(1), ..., p_i(l_i)\}$ for all $i \in \mathscr{L}$. Once more, we can represent the stochastic state of the system with a vector $X(v) = \{x_1(v_1), ..., x_a(v_a)\}$, where v_i represents the index of the capacity level of component $i \in \mathscr{L}$. We can use indifferently $X(v)$ or $v = \{v_i, ..., v_a\}$ to represent the state of G and its state-space as defined by Eq. 1. Furthermore, consider appropriate performance metric thresholds D_k for all $k \in K$ and the structure function $SF_k(v)$ that outputs 0 when $u_k(v) < D_k$ and 1 otherwise, where $u_k(v)$ measures the utility or performance of infrastructure $k \in K$. A simple extension of the structure function to the system-of-systems case is $SF(v) = \prod_{k \in K} SF_k(v)$. However, the simplicity of $SF(v)$ in this form comes with the drawback of needing input from an interdisciplinary body of experts that can establish acceptable performance level thresholds D_k for all infrastructures $k \in K$ in the aftermath of a crisis. We can estimate the systemic reliability $R(G)$ of the LSs' ensemble G as follows:

$$R(G) = \Pr[SF(v) = 1] = 1 - \Pr[SF(v) = 0] \tag{3}$$

Estimating $R(G)$ is also NP-Hard, since G can be reduced to the single infrastructure case. Note that while the findings of this study can be used in the single but larger infrastructure case, the reminder of this section is devoted to the computation of interdependent $R(G)$.

2.3 State-Space-Partition Methods for RAILS

The challenge of computing $R(G)$ resides in the high dimensionality of the problem ($a = |V| + |E|$) and the limited number of utility function evaluations that can be afforded with modern computational resources. When a is not too large, theoretical methods can be used to derive tight bounds on $R(G)$ as well as importance metrics to rank component importance [16]. On the other hand, when a is large, sampling methods seem to be the only option to LS modelers. Nevertheless, state-space-partition (SSP) methods can be used to derive sampling schemes that can be orders of magnitude faster than simulation-only methods [7, 13]. In addition to this, when evaluating $R(G)$ to different hazard scenarios such as in seismic risk assessment, decomposition methods prove even more profitable than simulation as previous decomposition results can be reutilized [20].

2.3.1 State-Space-Partition Methods

SSP methods are set-theoretic methods that generalize binary state decomposition methods (e.g. [8]) to the case of multi-state systems [2, 7]. The main strategy of SSP methods is to decompose Ω into disjoint sets of feasible and infeasible states. Before describing SSP methods in detail, we will first fix some notation. The infeasible domain $\mathscr{I} = \{v : SF(v) = 0\}$ can be written as the union of disjoint infeasible sets $I_i \in I$ enumerated using a SSP method. Similarly, the feasible domain $\mathscr{F} = \{v : SF(v) = 1\}$ can be expressed as the union of disjoint feasible sets $F_i \in F$ derived with an SSP method too. Furthermore, we can describe a hyper-rectangular subset $S \subseteq \Omega$ by its vertices $\alpha(S)$ and $\beta(S)$ such that $\alpha_i(S) \leq \beta_i(S)$ for all $i \in \mathscr{L}$. For brevity we use the notation $[\alpha(S), \beta(S)]$ to refer to the set $S = \{v : \alpha(S) \leq v_i \leq \beta(S), \forall i \in \mathscr{L}\}$. For example, we can describe the state space Ω by its vertices $\alpha(\Omega) = (1, .., 1)$ and $\beta(\Omega) = (l_1, .., l_a)$ which represent the states in which components are set to their minimum and maximum capacity levels, respectively. We can compute the probability $\Pr[v \in S]$ provided that S has a hyper-rectangular structure as follows:

$$\Pr[v \in S] = \prod_{i \in \mathscr{L}} \sum_{j=\alpha(S)}^{\beta(s)} p_i(j) \tag{4}$$

where $p_i(j)$ is the probability of component labeled $i \in \mathscr{L}$ being at capacity level $x_i(j)$ as defined in the previous subsection. Thus, for component capacity levels with valid PMFs, Eq. 4 yields $\Pr[v \in \Omega] = 1$. In general, SSP methods will decompose an input state-space U_{j-1} (for example, $U_0 = \Omega$) in two fashions.The feasible-based approach: $U_{j-1} = F_j \cup U_j$, where $U_j = U_{j-1} \setminus F_j$. Or, the infeasible-based approach: $U_{j-1} = I_j \cup U_j$, where, similarly, $U_j = U_{j-1} \setminus I_j$, and this represents an unexplored subset of Ω [7]. Also, F_j and I_j are determined by finding "deeper" states within U_{j-1} and leveraging on the properties of coherent systems. For the feasibility-based case assume that a system state v^0 is known such that $SF(v^0) = 1$ and $\alpha_i(U_{j-1}) \leq v_i^0 \leq \beta(U_{j-1})$ for all $i \in \mathscr{L}$. Then, a candidate for F_j is $[v^0, \beta(U_{j-1})]$ and we move such a set to the list of disjoint feasible sets F. We are left with U_j to be determined. Note that U_j is not guaranteed to be hyper-rectangular, however, we can partition U_j into disjoint sets U_i^j that have hyper-rectangular structure as shown in Eq. 5.

$$\begin{aligned} U_j^i = \{v \in \Omega : & v_k^0 \leq v_k \leq \beta_k(U_{j-1}) \text{ for } k < i, \\ & \alpha_i(U_{j-1}) \leq v_i < v_i^0 \text{ for } k = i, \\ & \alpha_k(U_{j-1}) \leq v_k \leq \beta_k(U_{j-1}) \text{ for } k > i\}, i \in \mathscr{L} \end{aligned} \tag{5}$$

An important remark from Eq. 5 is that for every SSP iteration there will be at most a subproblems U_j^i, however, some sets U_j^i may be empty. In particular, for any $i \in \mathscr{L}$ that $\alpha_i(U_j^i) = v_i^0$, Eq. 5 will output an empty set. Each non-empty subset U_j^i is moved to a list of disjoint unexplored subsets U. Next, we subtract a subproblem from the list U and use the decomposition process that was just outlined until U is empty, ergo Ω is fully partitioned. Note that at some iteration $j-1$ we may find no "deeper point" v^0 since the whole set U_{j-1} is infeasible. In such cases we shall move this set to the list of disjoint infeasible sets I. While carrying on this decomposition, the following bounds on systemic reliability are unraveled:

$$\sum_{F_i}^{F} \Pr[v \in F_i] \leq R(G) \leq 1 - \sum_{I_i}^{I} \Pr[v \in I_i] \tag{6}$$

Note that an infeasible-based partition follows symmetrically from the procedure outlined above. In practice, SSP methods will not always converge and thus the goal is to keep bounds in Eq. 6 sufficiently small. By prioritizing the exploration of subsets $U_i \in U$ based on their probability, this convergence can be accelerated; however, a processing scheme different from the Last-In-First-Out (LIFO) policy can result in a intractably large list of unexplored sets U. Thus, researchers have proposed to prioritize the selection of sets whenever possible and revert to a LIFO processing scheme if $|U|$ becomes too large [7]. We adopt this strategy in our computational experiments.

2.3.2 *An Improved State-Space-Partition Approach*

From the literature, there seems to be no definitive answer in regards to what SSP method will perform best given an arbitrary stochastic system G_k and threshold level D_k. When the performance threshold D_k is close to the maximum performance level of an infrastructure k, feasibility-based methods perform efficiently in practice [2]. On the other hand, when link reliabilities are low, empirical studies favor infeasible-based partitions as evidenced in the case of *s-t* network reliability [17]. This represents a challenge when selecting a SSP method that is effective for an ensemble of infrastructures with different reliabilities and performance thresholds. As an attempt to select a suitable SSP method for estimating systemic reliability, we propose a new approach that combines both, derivation of feasible and infeasible sets at the same time. We term the new approach Alternating State-Space-Partition (ASSP). Our approach begins by finding a state v^I such that $SF(v^I) = 0$ and such that raising any capacity level would cause the system-of-systems to become feasible. Then, we use analog version of equation Eq. 5 to find the complementary unexplored subsets. Moreover, for each unexplored subset, we remove a feasible set and find complementary unexplored subsets using equation Eq. 5. For an input subset U_{j-1} an ASSP algorithm consists of the following:

$$U_{j-1} = I_j \cup \bigcup_{i' \in \mathscr{L}} (F_j^i \cup \bigcup_{i \in \mathscr{L}} U_j^{(i',i)}) \tag{7}$$

Here, each iteration of an ASSP algorithm yields one infeasible set I_j, at most a feasible sets $F_j^{i'}$, and at most a^2 unexplored subsets $U_j^{(i',i)}$. The increased number of unexplored subsets can become an issue when sets are prioritized, however; it is still manageable by reverting to a LIFO decomposition policy if necessary. Another remark is better explained with a motivating example; consider the case of connectivity between nodes s and t in a single binary network system. Efficient algorithms to enumerate the list of independent paths can be found elsewhere. For each independent path we can reduce the capacity level of one link belonging to the path while keeping remaining links to their maximum capacity levels, hence we find a minimum cut-set (Menger's theorem). Then, for each unexplored subset generated using analog of Eq. 5, we can match an independent path solution and its respective path-set by keeping the capacity levels in the path to their maximum while all others are lowered to their minimum. Thus, in a single iteration of an ASSP algorithm we can compute at most $(1+a)$ subsets that contribute to shrinking bounds in Eq. 6 with little computational effort. This concept can be extended to feasible flow problems using the minimum-cut maximum-flow theorem [4]. Also, ASSP algorithms show remarkable improvement over strictly feasibility-based and strictly infeasible-based partitions as evidenced by our computational experiments in Section 4.

2.3.3 Importance and Stratified Sampling (ISS) Scheme

The performance of an SSP algorithm can be measured by the probability content in the unexplored sets in U. In practice, a SSP algorithm may not converge to the desired precision. In such cases, one can derive an estimator of $R(G)$ by taking n_i samples $v^k \in U_i$ for each unexplored subset $U_i \in U$, and estimate the probability of success when sampling from U_i as follows:

$$R_i(G) = \frac{\sum_{k=1}^{n_i} SF(v^k)}{n_i} \tag{8}$$

An estimator of $R(G)$ using this importance and stratified sampling (ISS) scheme is as follows:

$$R(\hat{G}) = \sum_{F_i \in F} \Pr[v \in F_i] + \sum_{U_i \in U} \Pr[v \in U_i] R_i(G) \tag{9}$$

We refer the reader to the literature for more details on this estimator[7]. When estimating $R(G)$ from Eq. 9 one can adopt Monte Carlo Sampling schemes with approximation guarantees [13]. We will used this estimator whenever the bounds in Eq. 6 do not converge to a desired precision.

2.3.4 Application Key Development: Feasible Flows in Interconnected Lifeline Systems

In this paper we will consider the problem of feasible multi-commodity flows in interconnected infrastructures $k \in K$ subject to node and link failures while thriving to guarantee performance levels D_k. Besides of the multi-commodity setting, this problem differs from the classical feasible flow reliability problem [2] because of the additional constraints on nodes of meeting certain demands in order to be functional [20]. More specifically, we will consider stochastic infrastructures $G_k(V_k, E_k)$ with V_k and E_k as defined above, and the system-of-systems ensemble $G(V, E)$ as defined above. Moreover, components have have capacity levels $u_{\psi(i)}$ for all nodes $i \in V_k$ and infrastructures $k \in K$, and capacity levels $u_{\phi(i,j)}$ for all links $(i, j) \in E_k$ and infrastructures $k \in K$. The one-to-one mapping functions ψ and ϕ map components to their labels $l \in \mathscr{L}$ remain as defined above. Furthermore, each infrastructure $k \in K$ has an associated set of commodities L_k and each node $i \in K$ has a demand $b_{i,l}$. When $b_{i,l_k} > 0$, we say that node $i \in V$ is a costumer of commodity $l_k \in L_k$. Conversely, when $b_{i,l_k} < 0$ we say that node $i \in V$ is a supplier of commodity $l_k \in L_k$, and a transshipment node otherwise. Certain nodes $i \in V$ will require satisfying demands $b'_{i,l_k} > 0$ to function properly and their capacity level will be factored by their functionalities $w_i \in \{0, 1\}$. In addition to this, flows $f_{(i,j),l_k}$ of commodity $l_k \in L_k$ can traverse links $(i, j) \in E_k \forall k \in K$. The favorable performance of the system-of-systems is verified when $d_k \geq D_k$ for all infrastructures $k \in K$, where d_k is the sum of flow of commodities $l_k \in L_k$ reaching costumer nodes. We can formulate the previous problem as a Mixed Integer Program (MIP); however, let us first consider the following reductions. Assume link costs $c_{i,j} = 0$ for all links $(i, j) \in E$. In order to maximize performance of the system-of-systems, for each infrastructure $G_k(V_k, E_k)$ introduce a super source node s_k and a super sink node t_k, and introduce fictitious link (t_k, s_k) with unbounded capacity $u_{\phi(t_k,s_k)} = \infty$ and negative cost $c_{t_k s_k} = -1$[1]. The set of labels $\mathscr{L}$ is extended with any newly added component. For every demand $b_{i,l_k} > 0$ and $b'_{i,l_k} > 0$ in node $i \in V$, we will introduce a fictitious link with deterministic capacity level $u_{\phi(i,t_k)} = b_{i,l_k}$ and null cost. Also, for every supply node $i \in V$, we will introduce a fictitious link with deterministic capacity level $u_{\phi(s_k,i)} = -b_{i,l_k}$ and null cost. The MIP formulation of this problem is shown in Eqs. 10. Despite being written as a minimization problem, the objective function in (10) maximizes the performance of G because of negative costs of links $(t_k, s_k) \in E_k, \forall k \in K$. The first set of constraints (10b) ensures flow balance at every node. The set of constraints (10c-d) ensures that the total amount of commodities does not exceed links' capacities while considering the functionality of end-nodes. The set of constraints (10e) ensures that the total amount of commodities traversing a node does not exceed its capacity level. The set of constraints (10f) ensures that the functionality of nodes is constrained by the satisfaction of demands b'_{il_k}. Finally, the set of constraints (10g) ensures non-negativity of flows.

$$\text{Min} \sum_{k\in K} \sum_{(i,j)\in E_k} \sum_{l_k\in L_k} c_{ij} f_{ijl_k} \tag{10a}$$

$$\text{s.t.} \sum_{j:(i,j)\in E_k} f_{ijl_k} - \sum_{j:(j,i)\in E_k} f_{jil_k} = 0, \qquad \forall i \in V, \forall l_k \in L_k, \forall k \in K \tag{10b}$$

$$\sum_{l_k\in L_k} f_{ijl_k} \leq u_{\phi(i,j)} w_i \qquad \forall (i,j) \in E_k, \forall k \in K \tag{10c}$$

$$\sum_{l_k\in L_k} f_{ijl_k} \leq u_{\phi(i,j)} w_j \qquad \forall (i,j) \in E_k, \forall k \in K \tag{10d}$$

$$\sum_{k\in K} \sum_{j:(i,j)\in E_k} \sum_{l_k\in L_k} f_{ijl_k} \leq u_{\psi(i)} w_i \qquad \forall i \in V \tag{10e}$$

$$w_i b'_{il_k} \leq f_{it_kl_k} \qquad \forall l_k \in L_k, \forall k \in K, \forall i \in V \tag{10f}$$

$$f_{ijl_k} \geq 0 \qquad \forall l_k \in L_k, \forall (i,j) \in E_k, \forall k \in K \tag{10g}$$

In the following sections, we will use the formulation in (10) to assess the performance of interconnected LSs and derive importance measures. For a more detailed description of the problem, its Mixed Integer Program (MIP) formulation, and how to use Eqs. 10 to derive "deeper" feasible or infeasible states we refer the reader to the work by the authors [20].

2.4 Importance Measures in RAILS

An attractive feature of non-simulation methods is that importance measures are readily available after partitioning a system into feasible and infeasible subsets with known probabilities. Here we provide approximations to importance measures in the literature [16] and extend them to the multi-state and systemic case based on the state-space partition of $R(G)$.

2.4.1 Systemic Reliability Sensitivity

Note that provided a full partition of Ω, Equations 6 and 4 provide polynomial functions in terms of capacity level probabilities $p_i(j)$ for all labeled components $i \in \mathscr{L}$ and capacity levels $\{1,..,l_i\}$. For simplicity, let us express those probabilities as p_{ij} with i and j as described above. Thus, we can compute partial derivatives of the polynomials to estimate component capacity level sensitivities. Note that the total probability theorem implies that, for every component, there is a dependent variable. Let us arbitrarily choose p_{i1} as the dependent variable. Thus, $p_{i1} = 1 - \sum_{j=2}^{l_i} p_{ij}$ and the sensitivity of capacity level reliabilities p_{ij} on systemic reliability $R(G)$ is as follows:

$$S_i = \sum_{j=2}^{l_i} \frac{\partial R(G)}{\partial p_{ij}} = \sum_{j=2}^{l_i} \sum_{F_k\in F} \frac{\partial \Pr[v \in F_k]}{\partial p_{ij}} = -\sum_{j=2}^{l_i} \sum_{I_k\in I} \frac{\partial \Pr[v \in I_k]}{\partial p_{ij}} \tag{11}$$

This metric evaluates the sensitivity of system reliability $R(G)$ with respect to overall reliabilities of component $i \in \mathscr{L}$.

2.4.2 Conditional Probability Importance Measure for Systemic Reliability

The second metric we consider is the conditional probability importance measure (CPIM)[22]. For the systemic reliability case, a labeled component $i \in \mathscr{L}$ at capacity level $j \in \{1,...,l_i\}$ has conditional probability importance measure $CPIM_{ij}$ as follows:

$$CPIM_{ij} = \Pr[v_i = j | SF(v) = 1] = \frac{\Pr[v \in \mathscr{F} \wedge v_i = j]}{R(G)} = \frac{\sum_{F_k\in F} \Pr[v \in F_k \wedge v_i = j]}{R(G)} \tag{12}$$

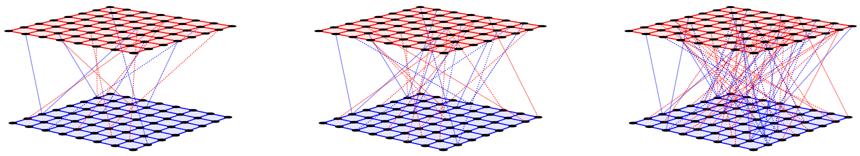

Figure 1: Realization of 2 8×8 grids with interdependency degrees I_{str} of 10%, 20% and 50%.

An estimator of the expression of the numerator in Eq. 12 can be obtained from Eq. 9 as follows:

$$\sum_{F_k \in F} \Pr[v \in F_k : v_i = j] + \sum_{U_i \in U} \Pr[v \in U_i] \frac{\sum_{k=1}^{n_i} SF(v^k) I(v_i^k = j)}{n_i} \tag{13}$$

where $I(v_i^k = j)$ is an indicator function that outputs 1 if $v_i^k = j$ and 0 otherwise. We adopt the importance measures generalized in this subsection in our computational experiments.

3 Computational Experiments

In this section we conduct computational experiments using the newly proposed ASSP method. In this study we will consider a set of synthetic networks with grid topology. Also, we will consider various performance threshold levels D_k and different scenarios of interconnectivity among infrastructure networks. We consider that both, links and nodes, can fail. For ease of interpretation we will adopt two capacity levels for components, namely the cases of complete damage and no-damage. Since we will need capacitated and interconnected systems with node demands, we will use the following network model. First, define the number of infrastructure grids $k \in K$ as well as their size $n \times n$. Note that whit this model, the ensemble contains $|K|(n^2 + 2(n-1)n)$ components. Later on, simulate intra-node usages for each infrastructure $k \in K$ identifying them as customers, suppliers, etc. Also, for each infrastructure $k \in K$ we select at random a percentage I_{str} of nodes and assign inter-demands b'_{i,l'_k} such that $k \neq k'$ and connect them to major consumer nodes j that are selected at random from infrastructure k' by adding a directed interdependent link (j,i) to E'_k with enough capacity to transport b'_{i,l'_k}. We selected as parameters 30% for the number of consumer and supplier nodes for each grid $k \in K$. Also, the intensity of all intra-demands and supplies are set to 1, maximum capacity level of links is set to 1 with probability 90%, and for nodes we adopt unbounded capacity with probability 95%. When components fail, their capacity levels are assumed to be 0. Figure 1 shows a realization of the model described above in which we have incrementally added inter-demands and interconnected links as I_{str} increases. In addition to the newly introduced ASSP method, for comparative purposes we will used strictly feasible-based partition (F-SSP) and strictly infeasible-based partitions (I-SSP). We used a high-performance cluster to conduct experiments in parallel. Each node of the cluster had a 12-core 2.83 GHz Intel Xeon processor, with 4GB of main memory, and each experiment was run on a single core. Each experiment consisted of running one of the three SSP algorithms on realizations of the model described above and used same magnitude of performance threshold D_k for each infrastructure $k \in K$. We set as limit of computation 1,000 seconds. Figure 2 showcases reliability estimates and associated variance obtained using the ASSP method. As we expected, when performance thresholds D_k increase we see a degradation of systemic reliability. Similarly, when interdependency levels increase we see further decrease of $R(G)$. From figure 2 we also note that there is a transition phase in the systemic reliability $R(G)$. Identifying such transitions in critical infrastructure networks would shed light on whether reducing interdependencies (by providing backup systems) or increasing system-capacity would be more cost-effective to shift the systemic reliability of the system to a safe level. A last remark from figure 2 is that in the transition region we verify an increase of variance on the $R(G)$ estimator. In our experiments, ASSP yielded the tightest

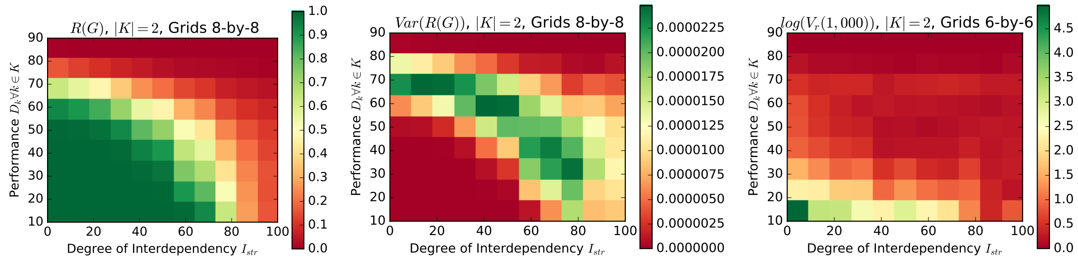

Figure 2: Systemic $R(G)$ for two 8×8 grids and associated variances for different D_k and I_{str} values using ASSP (Left and center). Logarithm (base 10) of the ratio of variance using FSSP over using ASSP.

bounds with respect the to other SSP methods. For small networks the variance reduction of ASSP with respect to other SSP methods was of several orders of magnitude (e.g. Fig. 2-Right). However, this trend reduced as the size of network increased while keeping the time threshold of 1,000 seconds constant. Also, for ensemble of grids above $10-by-10$ the gap $\Pr[v^k \in \mathscr{U}]$ was in the order of 0.7, and thus it resulted in very little gain in terms of variance reduction with respect to simulation.

4 Conclusions

In this paper we proposed an efficient analytical partitioning method termed the Alternating State Space Partition (ASSP) method and used an importance and stratified sampling (ISS) approximation scheme for systemic Reliability Assessment of Interdependent Lifeline Systems (RAILS). ASSP outperforms strictly feasible-based and strictly infeasible-based partitions. One of the main ideas in our ASSP approach is that of deriving large feasible and infeasible sets while solving one instance leveraging in the duality of the minimum-cut maximum flow theorem. In addition to this, we extended available importance measures in the literature to multi-state components and systemic reliability. The exact and theoretically bounded by-products of the RAILS framework contributes to measurement science in the context of infrastructure systems [12]. Further research should be devoted towards improving ASSP algorithms and testing with larger sets as well with real world benchmarks. Also, adding more system physical properties such as correlations, back up systems and recovery actions would augment the applications of this method. Finally, an in depth analysis of performance metrics that can be derived using SSP algorithms would definitely provide new perspectives of element ranking for protection and retrofitting in the context of interdependent systems and their resilience.

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